THE WEINBERG ANGLE IN A MAGNETIC FIELD AND THE RESTRICTION FOR A NEUTRINO MASS

V. A. GUSEINOV [a,b], R. E. GASIMOVA [a], E. Z. SAFAROVA [a]

a) Department of General and Theoretical Physics, Nakhchivan State University,AZ 7012, Nakhchivan, Azerbaijan;
b) Laboratory of Physical Research, Nakhchivan Division of Azerbaijan National Academy of Sciences, AZ 7000, Nakhchivan, Azerbaijan

The work is devoted to the calculation of the parameter $\sin^2\theta_w$ in an external magnetic field in the framework of the Weinberg-Salam-Glashow model. The new formula for $\sin^2\theta_w$ obtained from the ratio of the cross sections of the neutrino-electron and antineutrino-electron scattering reactions in a weak magnetic field shows that $\sin^2\theta_w$ is a function of the kinematic and dynamic parameters. The restriction for the neutrino (antineutrino) mass is found.

The existence of weak neutral currents was confirmed by the initial observation of $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$ reaction at CERN [1]. The relation between the weak coupling constant g and the electric charge e is established with the Weinberg angle θ_w, which characterizes the contribution of an electromagnetic current to a neutral current [2]

$$e = g\, sin\theta_w \tag{1}$$

where $g = 2^{4/5} G_F^{1/2} m_w$, G_F is the Fermi constant, m_w is the mass of the W boson.

On the basis of the experimental data on the neutral currents it was found that $sin^2\theta_w \approx 0.23 \pm 0.01$ [3]. This value of $\sin^2\theta_w$ belongs to $q^2 \approx m_z^{\,2}(m_w^2)$, where q^2 is the momentum transfer squar

$$sin^2\theta_w = \frac{e^2}{g^2} \tag{2}$$

is not constant and it is a function of the momentum transfer squared. One of the possibilities to determine $\sin^2\theta_w$ is measuring the ratio of the cross sections of the reactions [4-9]

$$\nu_l + e^- \to \nu_l + e^-, \tag{3}$$

$$\bar{\nu}_l + e^- \to \bar{\nu}_l + e^-, \tag{4}$$

where $\nu_l = \nu_\mu, \nu_\tau$ and $\bar{\nu}_l = \bar{\nu}_\mu, \bar{\nu}_\tau$.

Systematic uncertainties in the neutrino flux and spectrum create some difficulties in precise measurements of the cross sections of the considered processes. The primary uncertainty in the neutrino flux is connected with the uncertainty in the acceptance of the detector [8]. The questions concerning the limitation on the systematic uncertainties in the neutrino flux and spectrum is discussed in [3,7,8].

From the ratio of the cross sections of these reactions

$$Q \equiv \frac{\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)}{\sigma(\nu_\mu e \to \nu_\mu e)} \tag{5}$$

it has been found that $sin^2\theta_w = 0.209 \pm 0.032$ [5]. The CERN group reported the result $sin^2\theta_w = 0.211 \pm 0.037$ [6]. The agreement of these experimental results with the value of $\sin^2\theta_w$ predicted by the electroweak theory is one of the best confirmations of the Standard Model.

The reactions (3) and (4) in an external magnetic field can be used for testing the theory of electroweak interactions in a strong magnetic field. Measurement of the parameter $\sin^2\theta_w$ in an external magnetic field allows testing the Standard Model. This work is devoted to the calculation of $\sin^2\theta_w$ in an external magnetic field in the framework of the Weinberg-Salam-Glashow model. In this paper we also try to find the restriction for a neutrino mass.

Generally, electroweak radiative corrections are important for precise measurements of the cross sections of the considered processes. These corrections to neutrino-electron scattering in the Standard Model were computed by Sarantakos, Sirlin and Marciano [10]. Radiative corrections connected with leptonic interactions were also discussed in the review [11]. We perform our calculations in the tree level approximation. In this approximation and in the conditions of relatively low energies and weak magnetic fields the electroweak radiative corrections can be neglected. Of course, the consideration of these corrections can improve an agreement of the results of our calculations with the experimental data.

In our calculations we neglect neutrino masses and mixing. For generality, we take into account the propagator effects (Z boson contribution). But in the low-energy region we will use the low-energy approximation of the Standart Model.

In calculations of the cross sections of the neutrino (antineutrino)-electron scattering reactions we apply the method of the exact solution of the relativistic wave equation (the Dirac equation) in an external magnetic field. This method is one of the most effective methods of theoretical investigations and it allows to go out beyond the perturbation theory and foretell some theoretical predictions.

The cross section of the reaction (3) in a crossed electromagnetic field was found in [12]. It is well known that a constant crossed electromagnetic field simulates an arbitrary constant electromagnetic field of the strength

$$F << H_0 = \frac{m_e^2}{e} = 4.41 \times 10^{13}\, G \tag{6}$$

if ultra relativistic particles take part in the process. Here H_0 is the Schwinger field strength, e is the elementary charge and F can be the strength of a magnetic field or the strength of an electric filed.

Let us suppose that the electrons are ultra relativistic ($\varepsilon, \varepsilon' >> m_e$ where ε and ε' are the energies of the initial and final electrons and m_e is the electron mass) and the strength of a constant external electromagnetic field is $F << H_0$.

In this work we use the system of units where $\hbar = c = 1$. The signature of the metric is $(+---)$. As this is a constant external field we can consider the constant magnetic field. If we take into account that the vector of intensity of the constant magnetic filed H is directed along the axis Oz, the longitudinal momentum of the initial electron is zero and the vector of momentum of the initial neutrino (antineutrino) k is directed along H, we can write the following formulae for the invariant spectral variable u and the invariant parameters χ and κ:

$$u = \frac{\chi}{\chi'} - 1 = \frac{\varepsilon}{\varepsilon'} - 1, \qquad (7)$$

$$\chi = \frac{e}{m_e^3}\left[-(F_{\alpha\beta}p^\beta)^2\right]^{\frac{1}{2}} = \frac{\varepsilon}{m_e}\frac{H}{H_0}, \qquad (8)$$

$$\kappa = \frac{2(kp)}{m_e^2} = \frac{2k_0\varepsilon}{m_e^2} \qquad (9)$$

where $F_{\alpha\beta}$ is the tensor of the constant external field, $\chi' = \chi(p \to p')$, p and p' are the 4-momenta of the initial and final electrons, **k** is the 4-momentum of the initial neutrino (antineutrino), k_0 is the energy of the initial neutrino (antineutrino), H is the strength of an external magnetic field. Here χ is the dynamic parameter and κ is the kinematic parameter. In spite of the fact $(H/H_0) << 1$ due to the condition $\varepsilon >> m_e$ the field parameter χ can reach the value $\chi \geq 1$. In this case, the contribution of the field to the cross sections of the considered processes is significant.

In the framework of the Weinberg-Salam-Glashow model the cross sections of the processes (3) and (4) in a constant external magnetic field are

$$\sigma_\pm = \frac{G_F^2 m_e^2}{\pi^{3/2}}\int_0^\infty \frac{udu}{(1+u)^4}\left\{\left[\frac{\kappa}{u} f_\pm - 2g_L g_R(1+u)\right]\Phi_1(t) - 2\left[\frac{\chi}{u}\right]^{2/3} f_\pm \Phi'(t)\right\}\left[1+\kappa\left(\frac{m_e}{m_z}\right)^2\frac{u}{1+u}\right]^{-2} \qquad (10)$$

where m_z is the mass of a Z boson, σ_+ and $f_+ = g_L^2(1+u)^2 + g_R^2$ are for the reaction (3), σ_- and $f_- = g_R^2(1+u)^2 + g_L^2$ are for the reaction (4), $g_L = -\frac{1}{2} + \sin^2\theta_w$, $g_R = \sin^2\theta_w$. Here the functions

$$\Phi'(t) = \frac{d\Phi(t)}{dt} \qquad (11)$$

And

$$\Phi_1(t) = \int_t^\infty \Phi(\rho)d\rho \qquad (12)$$

are determined with the Airy function

$$\Phi(t) = (1/2\sqrt{\pi})\int_{-\infty}^{\infty} du\, exp\left[i(ty + \frac{1}{3}y^3)\right]. \qquad (13)$$

All these functions depend on a new variable

$$t = \left(\frac{u}{\chi}\right)^{\frac{2}{3}}\left(1 - \frac{\kappa}{u}\right). \qquad (14)$$

To determine $\sin^2\theta_w$ in an external magnetic field we find the ratio $R = \sigma_+/\sigma_-$:

$$R = \frac{(A_1 - B_1)g_L^2 + (A_2 - B_2)g_R^2 - 2g_L g_R C}{(A_1 - B_1)g_R^2 + (A_2 - B_2)g_L^2 - 2g_L g_R C} \qquad (15)$$

Where

$$\alpha_1 = A_1 - B_1, \alpha_2 = A_2 - B_2, \qquad (16)$$

$$A_1 = \kappa\int_0^\infty \frac{1}{(1+u)^2}\Phi_1 N du, \qquad (17)$$

$$A_2 = \kappa\int_0^\infty \frac{1}{(1+u)^4}\Phi_1 N du, \qquad (18)$$

$$B_1 = 2\chi^{\frac{2}{3}}\int_0^\infty \frac{u^{\frac{1}{3}}}{(1+u)^2}\Phi' N du, \qquad (19)$$

$$B_2 = 2\chi^{\frac{2}{3}}\int_0^\infty \frac{u^{\frac{1}{3}}}{(1+u)^4}\Phi' N du, \qquad (20)$$

$$C = \int_0^\infty \frac{u}{(1+u)^3}\Phi_1 N du. \qquad (21)$$

If we take into account $g_L = -\frac{1}{2} + \sin^2\theta_w$, $g_R = \sin^2\theta_w$, we obtain

$$R=\frac{A_1-B_1-4(A_1-B_1-C)\sin^2\theta_W+4(A_1-B_1+A_2-B_2-2C)\sin^4\theta_W}{A_2-B_2-4(A_2-B_2-C)\sin^2\theta_W+4(A_1-B_1+A_2-B_2-2C)\sin^4\theta_W}. \qquad (22)$$

According to the general theory developed in [13] the influence of the external magnetic field on the processes (3) and (4) is determined by the parameter

$$\eta=\frac{\chi}{\kappa}. \qquad (23)$$

Let us consider the asymptotic behaviour of the ratio R in the limiting case
$\eta<<1$ (weak field). For $\eta<<1$, we use the weak asymototic expansions of the Airy functions (see, for example, [14])

$$\Phi'(Ax)=\sqrt{\pi}A^{-2}\delta'(x)+O(A^{-5}), \qquad (24)$$

$$\Phi_1(Ax)=\sqrt{\pi}\left[\theta(-x)+\frac{1}{3}A^{-3}\delta''(x)+O(A^{-6})\right] \qquad (25)$$

where $\delta(x)=d\theta(x)/dx$ is the Dirac delta function, $A=\eta^{-2/3}$ is the parameter $(A>>1)$, $\theta(x)$ is the Heaviside function. Supposing in the expression (10) $\kappa(m_e/m_z)^2<<1$, we find the following expression for R:

$$R=\frac{ag_L^2+bg_R^2-cg_Lg_R+\frac{2}{s^3}\eta^2(dg_L^2+fg_R^2-hg_Lg_R)}{ag_R^2+bg_L^2-cg_Lg_R+\frac{2}{s^3}\eta^2(dg_R^2+fg_L^2-hg_Lg_R)} \qquad (26)$$

or

$$R=\frac{A_0\sin^4\theta_W-(A_0-B_0)\sin^2\theta_W+H_0+\frac{2}{s^3}\eta^2\left[D_0\sin^4\theta_W-(D_0-E_0)\sin^2\theta_W+I_0\right]}{A_0\sin^4\theta_W-B_0\sin^2\theta_W+C_0+\frac{2}{s^3}\eta^2(D_0\sin^4\theta_W-E_0\sin^2\theta_W+G_0)} \qquad (27)$$

where

$$A_0=a+b-c=4s^2-2s+1, \qquad (28)$$

$$B_0=b-\frac{1}{2}c=s^2-\frac{1}{2}s+1, \qquad (29)$$

$$C_0=\frac{1}{4}b=\frac{1}{4}(s^2+s+1), \qquad (30)$$

$$D_0=d+f-h=-2(s^4-6s^2+8s-5), \qquad (31)$$

$$E_0=f-\frac{1}{2}h=-s^3+3s^2-10s+10, \qquad (32)$$

$$G_0=\frac{1}{4}f=\frac{1}{4}(-3s^2-4s+10), \qquad (33)$$

$$H_0=\frac{1}{4}a=\frac{3}{4}s^2, \qquad (34)$$

$$I_0=\frac{1}{4}d=\frac{1}{4}s^2(-2s^2+2s+3) \qquad (35)$$

and

$$a=3s^2, \qquad (36)$$

$$b=s^2+s+1, \qquad (37)$$

$$c=3s, \qquad (38)$$

$$d=s^2(-2s^2+2s+3), \qquad (39)$$

$$f=-3s^2-4s+10, \qquad (40)$$

$$h=2s(s^2-6s+6). \qquad (41)$$

In the expressions (26-41) $s=\kappa+1=(q+p)^2/m_e^2$ is the normalized Mandelstam variable. The terms in (26) proportional to c and h correspond to the interference terms.In the expression (26) we retain the interference terms for low energy applications.

When the energy of the initial neutrino (antineutrino) is of order ~ MeV $(k_0=0.5MeV)$ and the energy of the electron is $\varepsilon=0.5GeV$, for the kinematic parameter κ we have $\kappa=2\times10^3>>1$ and $s=\kappa+1>>1$. In the weak magnetic field limiting case from (27) we find the following formula for $\sin^2\theta_w$:

$$\sin^2\theta_W=\frac{R-3+4\frac{\eta^2}{s}\pm\sqrt{-3R^2+10R-3+4\frac{\eta^2}{s}(R^2-6R+1)}}{8(R-1)(1-\frac{\eta^2}{s})}. \qquad (42)$$

From the last expression we see that the contribution of an external magnetic field to $\sin^2\theta_w$ is determined by the terms proportional to η^2/s. If we take into account the definition of η and formula for χ and κ, we have

$$\frac{\eta^2}{s}=\frac{1}{8}\left(\frac{H}{H_0}\right)^2\frac{m_e^4}{k_0^3\varepsilon}. \tag{43}$$

The contribution η^2/s is determined by three variables k_0, H and ε. This contribution is especially significant when we have dealings with relatively strong magnetic field and low-energy neutrinos (antineutrinos). In this case not very high ε is desirable. If we set $H=10^{-2}H_0$, $k_0=0.5MeV$ and $\varepsilon=0.5GeV$, the estimations give $\eta^2/s=0.625\times10^{-8}$ that is very small.

According to the formula (43) the upper limit of η^2/s is determined by the maximal possible value of H and the minimal possible values of k_0 and ε. The order of the maximal value of η^2/s can not be greater than $\sin^2\theta_w=0.23\sim10^{-1}$. The maximal possible value for H can be $H\sim H_0$ and the minimal possible value for the electron energy can be $\varepsilon\sim m_e$. In this case the value $\sin^2\theta_w\sim10^{-1}$ can be achieved due to the multiplier k_0^{-3}.

However, there is definite restriction for the lowest value k_{0min} of the neutrino (antineutrino) energy k_0. The neutrino (antineutrino) energy k_0 can not be smaller than the value k_{0min}. Otherwise, the order of the parameter $\sin^2\theta_w$ would be greater than $\sin^2\theta_w\sim0.1$. But η^2/s can not be greater than $\sim10^{-1}$. It means that the lowest energy of a neutrino (antineutrino) is k_{0min}. If a neutrino (antineutrino) is a massive particle, the lowest limit of k_0 corresponds to neutrino (antineutrino) mass. So, the muon neutrino (antineutrino) mass m_{ν_l} can not be greater than k_{0min}:

$$m_{\nu_l}\le k_{0min}=\frac{1}{2}\left[\frac{\left(\frac{H}{H_0}\right)^2\frac{m_e^4}{\varepsilon}}{\frac{\eta^2}{s}}\right]^{1/3}\approx\frac{1}{2}m_e$$

or $$m_{\nu_l}\le0.255MeV. \tag{44}$$

This estimation is right for ν_μ, $\bar\nu_\mu$, ν_τ and $\bar\nu_\tau$.

---

[1] *Hasert.F et al* (Gargamelle Collaboration) 1973 Phys. Lett. **B 46** 121
[2] *Okun.L* B 1985 *Leptons and Quarks* (Amsterdam: North-Holland)
[3] *See Panman J* 1995 *Precision Tests of the Standard Electroweak Model* edP. Langacker (Singapore: World Scientific) p 504 and references therein
[4] *Steinberger J* 1990 Uspekhi Fizicheskikh Nauk **160** 136
[5] *Ahrens L A et al* 1990 *Phys. Rev.* D **41** 3297
[6] Dorenbasch J *et al* 1989 *Z. Phys.* C **41** 567
[7] *Marciano W J* and *Parsa Z* 2003 J. Phys. G **29** 2629
[8] *Imlay R* and *VanDalen G J* 2003 J. Phys. G **29** 2647
[9] *Vilain P et al* 1994 Phys. Lett. **B 335** 246
[10] *Sarantakos S, Sirlin A* and *Marciano W* J Nucl. Phys. **B 217** 84
[11] *Borisov A V, Vshivtsev A S, Zhukovskii V Ch* and *Eminov P A* 1997 Uspekhi Fizicheskikh Nauk **167** 241
[12] *Borisov A V, Nanaa M K* and *Ternov I M* 1993 Vestnik Mosk. Univ. Ser. 3 Fizika. Astronomiia **2** 15
[13] *Ritus V I* and *Nikishov A I* 1979 Trudy FIAN SSSR **111**(Moscow: Nauka)
[14] *Borisov A V, Guseinov V A* and *Zamorin N B* 2000 Phys. At. Nuclei **63** 1949
[15] *Borisov A V, Guseinov V A* and *Zamorin N B* 2000 Yad. Fiz. **63** 2041

В. А. ГУСЕЙНОВ, Р. Э. ГАСЫМОВА, Э. З. САФАРОВА

УГОЛ ВАЙНБЕРГА В МАГНИТНОМ ПОЛЕ И ОГРАНИЧЕНИЕ НА МАССУ НЕЙТРИНО

Работа посвящена вычислению параметра $\sin^2\theta_w$ в рамках модели Вайнберга-Салама-Глешоу во внешнем магнитном поле. Новая формула для $\sin^2\theta_w$, полученная из соотношения сечений реакций нейтрино-электронной и антинейтрино-электронной рассеяний в слабом магнитном поле, показывает, что $\sin^2\theta_w$ является функцией кинематического и динамического параметров. Найдено ограничение на массу нейтрино (антинейтрино).

V. A. HÜSEYNOV, R. E. QASIMOVA, E. Z. SƏFƏROVA

MAQNİT SAHƏSİNDƏ VAYNBERQ BUCAĞI VƏ NEYTRİNO KÜTLƏSİ ÜÇÜN MƏHDUDİYYƏT

İş xarici maqnit sahəsində Vaynberq-Salam-Qleşou modeli çərçivəsində $\sin^2\theta_w$ parametrinin hesablanmasına həsr olunmuşdur. Zəif maqnit sahəsində neytrino-elektron və antineytrino-elektron səpilmə reaksiyalarının en kəsiklərinin nisbətindən $\sin^2\theta_w$ üçün alınmış yeni ifadə göstərir ki, $\sin^2\theta_w$ kinematik və dinamik parametrlərin funksiyasıdır. İşdə neytrino (antineytrino) kütləsi üçün məhdudiyyət tapılmışdır.